J. Fluid Mech. (1978), *vol.* 84, *part* 1, *pp.* 207–208
Printed in Great Britain

REVIEW

Quasilinear Hyperbolic Systems and Waves. By A. Jeffrey. Pitman, 1976. 230 pp. £6.90.

One of the most familiar examples of nonlinear wave propagation governed by a quasi-linear hyperbolic equation is provided by one-dimensional disturbances of finite amplitude in a polytropic gas. Here one already encounters the typical features of change of the wave profile, the development of singularities of classical (that is, continuously differentiable) solutions, and the occurrence of shock waves. The theory goes back to the nineteenth century; Riemann's seminal paper was written in 1859, and shocks were first treated by Hugoniot and Rankine. During the war there was considerable activity in this field, and a classic account of the subject as it stood in the mid 1940s is given in the well-known book by Courant and Friedrichs. Since then, the theory of wave propagation governed by quasi-linear hyperbolic equations has developed rapidly. The literature is now very extensive, and the stream of papers shows little sign of diminishing. So it is no easy task for a newcomer to the subject to familiarize himself with the literature, or for an interested outsider to get some idea of the thrust of recent developments. Professor Jeffrey has written a very readable and accessible account which should be of material help. By staying within the framework of classical analysis, and including detailed introductory material, he has produced a monograph which, without being unduly long, can serve both as a textbook and as an introduction to current research. He has, rightly, confined his exposition mainly to quasi-linear systems in one space variable and time; this is the most highly developed part of the subject at present. The emphasis in the book is on the analysis of the transport of discontinuities along characteristics (which correspond to wave fronts), on the effect of nonlinearity which causes classical solutions to break down, and on the way in which such solutions can be extended by introducing shock waves when the governing equations represent conservation laws and have a right-hand side in divergence form. The theoretical development is copiously illustrated by detailed physical examples on such topics as gas flow and shallow-water waves.

There are five chapters. The first two, on 'Nonlinear equations and quasilinear systems' and on 'Hyperbolic systems and characteristics', are introductory, and could be the basis of a useful course at third-year Honours level. The main matter of the book begins with chapter 3, on (generalized) 'Riemann invariants and simple waves'; both of these concepts are familiar in the theory of one-dimensional gas motion, and they have now been generalized extensively. Chapter 4, on 'Shock waves', introduces the reader to weak solutions, one of the key ideas. This makes it possible to treat discontinuous solutions (shocks) in a very general context, but leads to a failure of uniqueness which must be made good by introducing a principle for selecting a physically meaningful solution. This is derived here from the so-called evolutionary condition, which is the requirement that a shock must be stable to small disturbances. The last chapter, 'Development of shocks from Lipschitz continuous data', is largely based on the author's own contributions to this important problem; it is illustrated by a detailed analysis of shallow-water waves.

The book has an index, and a bibliography listing over 140 papers and books. As this includes many references to work that had to be omitted for reasons of space or (in some cases) its more elaborate mathematical apparatus, it should prove a most useful adjunct for the interested reader.

F. G. FRIEDLANDER